

RADIOACTIVITY

Radioactivity is a nuclear phenomenon in which an unstable nucleus undergoes a decay. This is referred to as radioactive decay. Three types of radioactive decay occur in nature :

- (i) α -decay in which a helium nucleus He_4 is emitted;
- (ii) β -decay in which electrons or positrons (particles with the same mass as electrons, but with a charge exactly opposite to that of electron) are emitted;
- (iii) γ -decay in which high energy (hundreds of keV or more) photons are emitted.

Law of radioactive decay

If N is the number of nuclei in the sample and ΔN undergo decay in time Δt then

$$\Delta N / \Delta t \propto N \quad \text{or,} \quad \Delta N / \Delta t = \lambda N,$$

where λ is called the radioactive decay constant or disintegration constant.

Thus the rate of change of N is (in the limit $\Delta t \rightarrow 0$)

$$\frac{dN}{dt} = -\lambda N \quad \text{or,} \quad \frac{dN}{N} = -\lambda dt$$

Now, integrating both sides of the above equation, we get,

$$\int_{N_0}^N \frac{dN}{N} = -\lambda \int_{t_0}^t dt$$

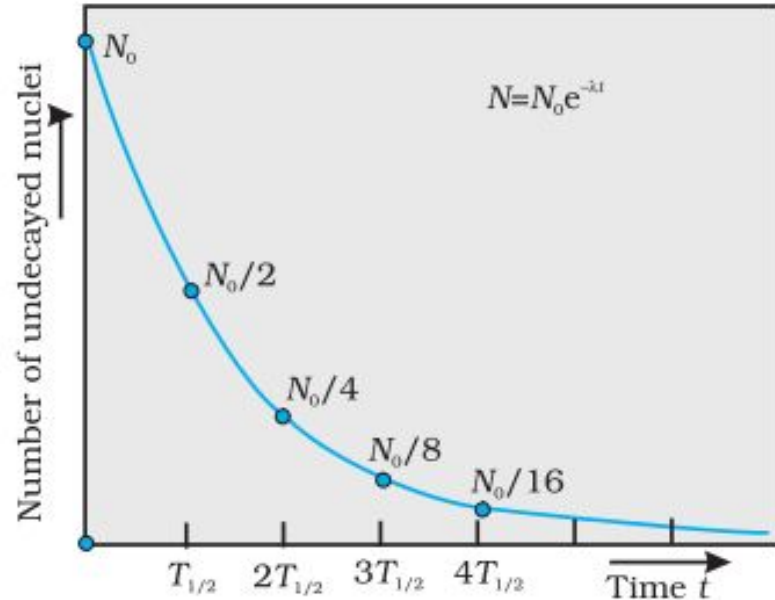
$$\text{or, } \ln N - \ln N_0 = -\lambda (t - t_0)$$

Here N_0 is the number of radioactive nuclei in the sample at some arbitrary time t_0 and N is the number of radioactive nuclei at any subsequent time t .

$$\ln \frac{N}{N_0} = -\lambda t$$

which gives

$$N(t) = N_0 e^{-\lambda t}$$



The total decay rate R of a sample is the number of nuclei disintegrating per unit time.

The positive quantity R is then defined as

$$R = -\frac{dN}{dt}$$

$$R = \lambda N_0 e^{-\lambda t}$$

$$\text{or, } R = R_0 e^{-\lambda t}$$

The decay rate R at a certain time t and the number of undecayed nuclei N at the same time are related by $R = \lambda N$

1 becquerel is simply equal to 1 disintegration or decay per second.

There is also another unit named “curie” that is widely used and is related to the SI unit as:

$$\begin{aligned} 1 \text{ curie} &= 1 \text{ Ci} = 3.7 \times 10^{10} \text{ decays per second} \\ &= 3.7 \times 10^{10} \text{ Bq} \end{aligned}$$

After a lapse of $T_{1/2}$, population of the given species drops by a factor of 2.

Half-life of a radionuclide (denoted by $T_{1/2}$) is the time it takes for a sample that has initially, say N_0 radionuclei to reduce to $N_0/2$. Putting

$N = N_0/2$ and $t = T_{1/2}$

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

average or mean life τ

Another related measure is the average or mean life τ .

The number of nuclei which decay in the time interval t to $t+\Delta t$ is

$$R = \lambda N$$

ie $R(t)\Delta t (= \lambda N_0 e^{-\lambda t} \Delta t)$.

To obtain the mean life, we have to sum (or integrate) this expression over all times from **0 to ∞** , and divide by the total number N_0 of nuclei at $t = 0$. Thus,

$$\tau = \frac{\int_0^{\infty} \lambda N_0 t e^{-\lambda t} dt}{N_0} = \lambda \int_0^{\infty} t e^{-\lambda t} dt$$

One can show by performing this integral that $\tau = 1/\lambda$

We summarise these results with the following:

$$T_{1/2} = \frac{\ln 2}{\lambda} = \tau \ln 2$$