POTENTIAL ENERGY IN AN EXTERNAL FIELD

Potential energy of a single charge

The external field E is not produced by the given charge(s) whose potential energy we wish to calculate. E is produced by sources external to the given charge(s).

We **assume** that the charge q does not significantly **affect** the sources producing the external field. This is true if q is **very small**

By definition, V at a point P is the work done in bringing a unit positive charge from infinity to the point P.

Thus, **work done** in bringing a **charge q** from **infinity** to the point **P** in the external field is qV.

This **work** is **stored** in the form of **potential energy** of **q**. If the point P has position vector r relative to some origin, we can write:

Potential energy of q at r in an external field

$$\mathbf{U = qV(r)}$$

where V(r) is the external potential at the point r.

Thus, if an electron with charge $q = e = 1.6\times10^{-19}$ C is accelerated by a potential difference of 1 volt, it would gain energy of $eV = 1.6\times10^{-19}$ J.

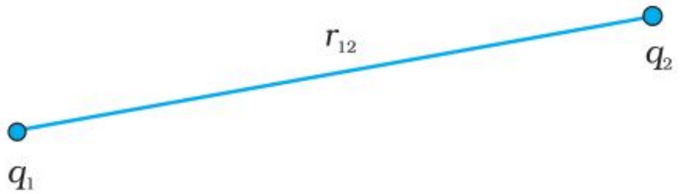
This unit of energy is defined as 1 electron volt or 1eV, i.e., **$1\ eV = 1.6\times10^{-19}$ J**.

Potential energy of a system of two charges in an external field

Consider first the simple case of two charges q_1 and q_2 with position vector r_1 and r_2 relative to some origin.

There is **no external field** against which work needs to be done, so **work done** in bringing q_1 from infinity to r_1 is **zero**.

The charge q_1 produces a potential in space given by



$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{1P}}$$

From the definition of potential, work done in bringing charge q_2 from infinity to the point r_2 is q_2 times the potential at r_2 due to q_1:

$$\text{work done on } q_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$$

Since electrostatic force is conservative, this work gets stored in the form of potential energy of the system. Thus, the potential energy of a system of two charges q_1 and q_2 is

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$$

Let us calculate the potential energy of a system of three charges q1, q2 and q3 located at r1, r2, r3, respectively. To bring q1 first from infinity to r1, no work is required. Next we bring q2 from infinity to r2. As before, work done in this step is

$$q_2 V_1(\mathbf{r}_2) = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_{12}}$$

The charges q1 and q2 produce a potential, which at any point P is given by

$$V_{1,2} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_{1P}} + \frac{q_2}{r_{2P}}\right)$$

Work done next in bringing q_3 from infinity to the point r_3 is q_3 times $V_{1,2}$ at r_3

$$q_3 V_{1,2}(\mathbf{r}_3) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

The total work done in assembling the charges at the given locations is obtained by adding the work done in different steps

$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

POTENTIAL ENERGY IN AN EXTERNAL FIELD

For a single charge

Potential energy of q at r in an external field **U = qV(r)**

where V(r) is the external potential at the point r.

For a system of two charges in an external field

Work done on q_2 against the external field

$= q_2V(r_2)$

Work done on q2 against the field due to q1

$$= \frac{q_1 q_2}{4\pi\varepsilon_o r_{12}}$$

By the superposition principle for fields, we add up the work done on q2 against the two fields (E and that due to q1):

Work done in bringing q2 to r2

$$= q_2 V(\mathbf{r}_2) + \frac{q_1 q_2}{4\pi\varepsilon_o r_{12}}$$

Thus, Potential energy of the system

= the total work done in assembling the configuration

$$= q_1 V(\mathbf{r}_1) + q_2 V(\mathbf{r}_2) + \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}$$

Potential energy of a dipole in an external field

Consider a dipole with charges $q_1 = +q$ and $q_2 = -q$ placed in a uniform electric field E, Then above equation becomes in a uniform electric field, the dipole experiences no net force; but experiences a torque τ given by $\tau = p \times E$ which will tend to rotate it

If **2a cosθ** is the parallel distance between two charges, the Potential difference

$U(\theta) = [V(r_1) – V(r_2)] = –E \times 2a \cos\theta$

or $q[V(r_1) – V(r_2)] = –E \times (q x\ 2a) \cos\theta$

$= -p.E$

Therefor we can obtain

$$U'(\theta) = -pE\cos\theta - \frac{q^2}{4\pi\varepsilon_0 \times 2a} = -\mathbf{p}\cdot\mathbf{E} - \frac{q^2}{4\pi\varepsilon_0 \times 2a}$$