KINETIC THEORY OF AN IDEAL GAS

Collission of molecules

The molecules collide incessantly against each other or with the walls and change their velocities. The collisions are considered to be elastic. Momentum and KE are conserved in collisions

The medium is considered to be isotropic and the molecules are in Random Motion.

One dimensional case (x direction)

Consider elastic collision of the molecule in a wall of area A (l^2).

Change in momentum of the molecule

$$-mv_x - (mv_x) = -2mv_x$$

The momentum imparted on the wall in **each collision** is

$$+2mv_x$$

The total momentum transferred to the wall

In a small time interval Δt, a molecule with x-component of velocity v_x will hit the wall if it is ***within the distance*** **$v_x \Delta t$** from the wall.

All molecules within the **volume $A v_x \Delta t$** only can hit the wall in time Δt.

If n is the number of molecules per unit volume

$Av_x \Delta t\, n$ molecules may hit the wall

For an isotropic medium, half of the molecules may in positive X - direction and other half may in negative X direction.

That is $\frac{1}{2} A v_x \Delta t\, n$ molecules hit the wall in Δt seconds

The total momentum transferred to the wall by these molecules in time Δt is : $Q = (2mv_x)(1/2\ n\ A\ v_x\ \Delta t)$

As **Force** is the rate of change of momentum,

$$F = Q / \Delta t = (2mv_x)(1/2\ n\ A\ v_x) = n\ m\ v_x^2\ A$$

Then Pressure $P = F / A = n\ m\ v_x^2$

If $\overline{v_x^2}$ is the average of v_x^2

$$P = nm\overline{v_x^2}$$

For an isotropic medium in 3 dimensions

Mean squared velocity $\overline{v^2} = \overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2} = 3\overline{v_x^2}$

Since

$$\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2}$$

Then

$$\overline{v_x^2} = \frac{\overline{v^2}}{3}$$

Pressure of an Ideal Gas

Then the total Pressure of an Ideal Gas

$$P=\left(\frac{1}{3}\right)nm\bar{v}^2$$

Kinetic Interpretation of Temperature

Multiplying above equation with Volume

$$PV = \left(\frac{1}{3}\right)(nV)mv^2 = \left(\frac{1}{3}\right)N\,mv^2 = \left(\frac{2}{3}\right)N\left(\frac{1}{2}mv^2\right)$$

Since the internal energy E of an ideal gas is purely kinetic

$$E = N\left(\frac{1}{2}mv^2\right) = N\,(KE)$$

$$PV = \left(\frac{2}{3}\right)E$$

Cross multiplying

$$E = \left(\frac{3}{2}\right)PV = \left(\frac{3}{2}\right)\mu RT$$

$$E = \left(\frac{3}{2}\right)PV = \left(\frac{3}{2}\right)\mu RT = \left(\frac{3}{2}\right)\mu (N_A k_B)T = \left(\frac{3}{2}\right)N k_B T$$

Average kinetic energy per molecule

$$\frac{E}{N} = \left(\frac{3}{2}\right)k_B T$$

The average kinetic energy of a molecule is **proportional** to the **absolute temperature** of the gas; it is ***independent*** of ***pressure, volume or the nature*** of the ideal gas.