SPECIFIC HEAT CAPACITY

Heat Capacity

$$S = \frac{\Delta Q}{\Delta T}$$

And it is proportional to the mass of the substance

Specific heat capacity

$$s = \frac{S}{m} = \frac{\Delta Q}{m \Delta T}$$

It depends on the nature of the substance and its temperature. The unit of specific heat capacity is J kg^{-1} K^{-1}

Molar specific heat capacity

$$C = \frac{S}{\mu} = \frac{\Delta Q}{\mu \Delta T}$$

Molar specific heat capacity is **independent of the amount of substance**. It **depends on the nature of the substance**, its **temperature** and the conditions under which heat is supplied. The unit of C is J mol $^{-1}$ K $^{-1}$.

Equipartition of Energy

In equilibrium, the total energy is equally distributed in all possible energy modes, with each mode having an average energy equal to **$1/2\ k_B T$**

Since a **vibrational** mode has both **kinetic** and **potential** energy modes, each solid molecule possess average energy of **$2 \times 1/2\ k_B T = k_B T$**

In three dimensions, the average energy is $3k_BT$

For one mole of a solid

Internal energy $\mathbf{U = N_A\ 3k_BT = 3\ RT}$

Where $\mathbf{R = N_A\ k_B}$ is the Universal gas constant

Pressure remains constant for a Solid

since $\Delta V = 0$, $\Delta Q = \Delta U + P\,\Delta V \cong \Delta U$

That is specific heat capacity for one mole of solid

$$C = \frac{\Delta Q}{\Delta T} = \frac{\Delta U}{\Delta T} = 3R$$

This agreement is known to break down at **low temperatures**

Specific heat capacity of water

With more precise measurements, it was found that the specific heat of water varies slightly with temperature.

One calorie is defined to be the amount of heat required to raise the temperature of 1g of water from 14.5 °C to 15.5 °C

Specific heat capacity of Gases

For gases, for example, we can define two specific heats : specific heat capacity at constant volume and specific heat capacity at constant pressure.

For ideal gas, we have a simple relation $\mathbf{C_p - C_v = R}$

where Cp and Cv are molar specific heat capacities of an ideal gas at constant pressure and volume respectively and R is the universal gas constant.

At constant volume Cv

We have ∆Q = ∆U + P ∆V

If ∆Q is absorbed at constant volume, ∆V = 0, then ∆Q = ∆U

$$C_v = \left[\frac{\Delta Q}{\Delta T}\right]_v = \left[\frac{\Delta U}{\Delta T}\right]_v = \frac{dU}{dT}$$

∆U depends on the temperature only

At constant pressure Cp

Gas must do some additional work to maintain constant pressure

$$C_p = \left[\frac{\Delta Q}{\Delta T}\right]_p = \left[\frac{\Delta U}{\Delta T}\right] + P\left[\frac{\Delta V}{\Delta T}\right]_p$$

That is **Cp > Cv**

Or

$$C_p - C_v = P\left[\frac{\Delta V}{\Delta T}\right]_P$$

But we have **PV = RT** for **one mole** of gas

For constant Pressure **$P\,\Delta V = R\,\Delta T$**

$$P\left[\frac{\Delta V}{\Delta T}\right]_P = R$$

Clearly C_p-C_v = R **It is the Mayor's Relation**