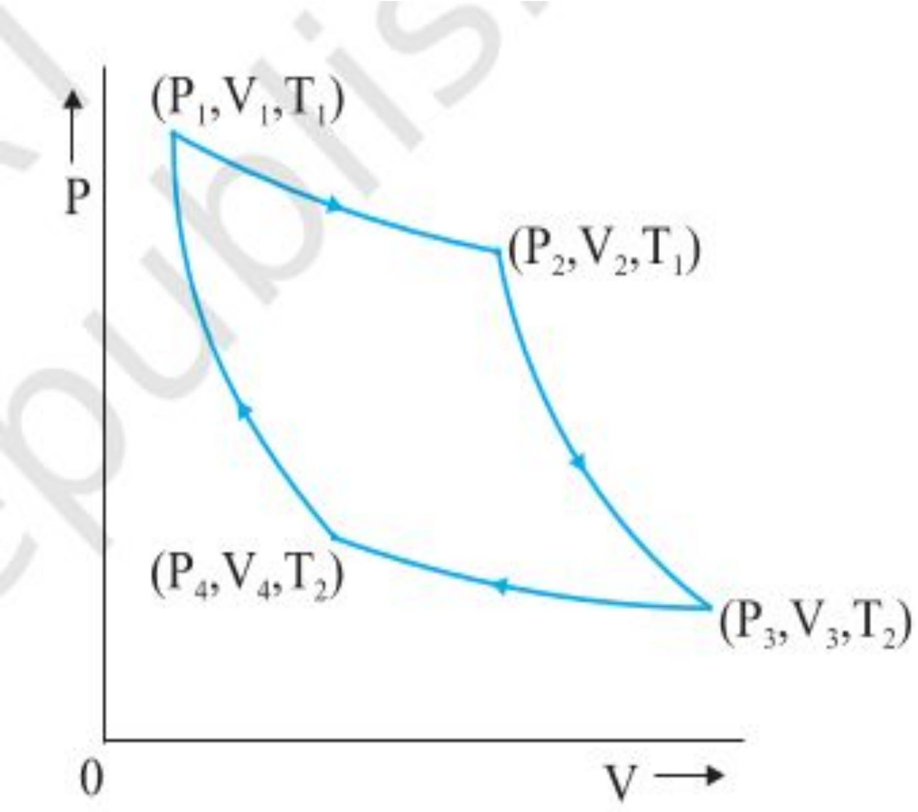
CARNOT ENGINE

We expect the ideal engine operating between two temperatures to be a reversible engine

Suppose we have a hot reservoir at temperature T_1 and a cold reservoir at temperature T_2



A reversible heat engine operating between two temperatures is called a Carnot engine.

We have taken the working substance of the Carnot engine to be an ideal gas.

Isothermal expansion

Isothermal expansion of the gas taking its state from

($\mathbf{P_1}$, $\mathbf{V_1}$, $\mathbf{T_1}$) to ($\mathbf{P_2}$, $\mathbf{V_2}$, $\mathbf{T_1}$).

The heat absorbed by the gas (Q_1) from the reservoir at temperature T_1

Work done in Isothermal Expansion

Work done **by the gas** on the environment.

$$W_{1\to 2}=Q_1=\mu RT_1\ln\left[\frac{V_2}{V_1}\right]$$

Adiabatic expansion

Adiabatic expansion of the gas from

$(\mathbf{P}_2, \mathbf{V}_2, \mathbf{T}_1)$ to $(\mathbf{P}_3, \mathbf{V}_3, \mathbf{T}_2)$

Work done **by the gas**

$$W_{2\to3} = \left[\frac{\mu R (T_1 - T_2)}{\gamma - 1}\right]$$

Isothermal compression

Isothermal compression of the gas from

(P_3 , V_3 , T_2) to (P_4 , V_4 , T_2).

Work done **on the Gas**

$$W_{3\to4}=Q_2=\mu R T_2 \ln\left[\frac{V_3}{V_4}\right]$$

Adiabatic compression

Adiabatic compression of the gas from

(P_4, V_4, T_2) to (P_1, V_1, T_1)

Work done **on the gas**

$$W_{4\to1} = \left[\frac{\mu R(T_1 - T_2)}{\gamma - 1}\right]$$

Total work done

Total work done by the gas in one complete cycle is given by

$$W = W_{1 \to 2} + W_{2 \to 3} - W_{3 \to 4} - W_{4 \to 1}$$

$$W = \mu R T_2 \ln\left[\frac{V_3}{V_4}\right] - \mu R T_2 \ln\left[\frac{V_3}{V_4}\right]$$

Efficiency

$$\eta = 1 - \frac{Q_2}{Q_1}$$

And

$$\eta = \frac{W}{Q_1} = \frac{\mu R T_1 \ln\left[\frac{V_3}{V_4}\right] - \mu R T_2 \ln\left[\frac{V_3}{V_4}\right]}{\mu R T_1 \ln\left[\frac{V_3}{V_4}\right]}$$

On simplification

$$\eta = 1 - \frac{T_2 \ln\left[\frac{V_3}{V_4}\right]}{T_1 \ln\left[\frac{V_2}{V_1}\right]}$$

We can find that $\left[\frac{V_3}{V_4}\right] = \left[\frac{V_2}{V_1}\right]$ In the following way

In the Adiabatic Expansion Process (P_2 , V_2 , T_1) to (P_3 , V_3 , T_2)

$$P_2 V_2^{\gamma} = P_3 V_3^{\gamma}$$

But $P_2 V_2 = \mu RT_1$ and $P_3 V_3 = \mu RT_2$

$$\mu RT_1 V_2^{\gamma-1} = \mu RT_2 V_3^{\gamma-1}$$

$$T_1 V_2^{\gamma-1} = T_2 V_3^{\gamma-1}$$

Cross Multiplying

$$\left[\frac{V_2}{V_3}\right] = \left[\frac{T_2}{T_1}\right]^{\frac{1}{(\gamma-1)}}$$

Similarly in the adiabatic compression process

($\mathbf{P_4}$, $\mathbf{V_4}$, $\mathbf{T_2}$) to ($\mathbf{P_1}$,$\mathbf{V_1}$, $\mathbf{T_1}$)

$$\left[\frac{V_1}{V_4}\right] = \left[\frac{T_2}{T_1}\right]^{\frac{1}{(\gamma-1)}}$$

On Comparison

$$\left[\frac{V_2}{V_3}\right]=\left[\frac{V_1}{V_4}\right]$$ Or $$\left[\frac{V_2}{V_1}\right]=\left[\frac{V_3}{V_4}\right]$$

Then we prove that $$\eta=1-\frac{T_2}{T_1}$$

In Carnot Cycle

$$\frac{T_2}{T_1} = \frac{Q_2}{Q_1}$$

It is a universal relation independent of the nature of the system