

INDUCTANCE

An electric current can be induced in a coil by flux change produced by another coil in its vicinity or flux change produced by the same coil.

In both the cases, the flux through a coil is proportional to the current.

That is, $\Phi_B \propto I$

When the flux Φ_B through the coil changes, each turn contributes to the induced emf.

Therefore, a term called **flux linkage** is used which is equal to $N\Phi_B$ for a closely wound coil with N turns and in such a case

$$N\Phi_B \propto I$$

The constant of proportionality, in this relation, is called **inductance**

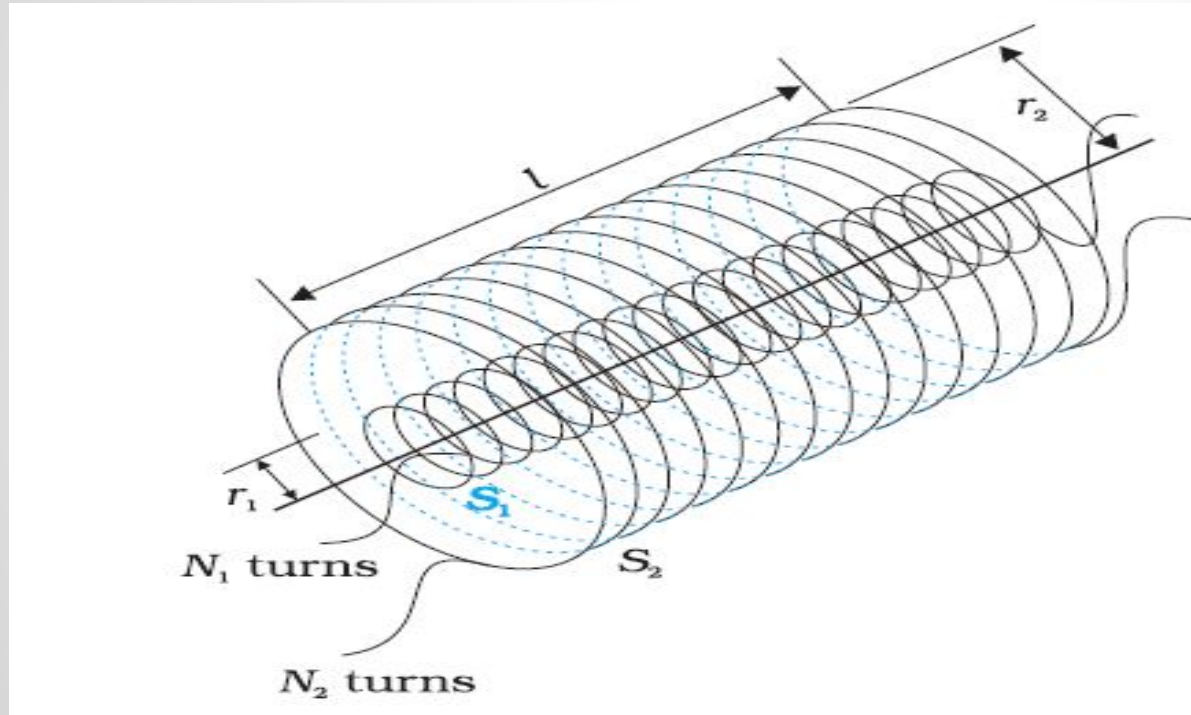
The inductance depends only on the geometry of the coil and intrinsic material properties. Inductance is a scalar quantity

The SI unit of inductance is henry and is denoted by H

if the geometry of the coil does not vary with time then

$$\frac{d\Phi_B}{dt} \propto \frac{dI}{dt}$$

Mutual inductance



When a current I_2 is set up through S_2 , it in turn sets up a magnetic flux through S_1 . Let us denote it by Φ_1 .

The corresponding flux linkage

with solenoid S_1 is $N_1 \Phi_1 \propto I_2$

or $N_1 \Phi_1 = M_{12} I_2$ M_{12} is called the mutual inductance of solenoid S_1 with respect to solenoid S_2 .

It is also known as the **coefficient of mutual Induction**

But $N_1 \Phi_1 = (n_1 l) (B_2 A_1)$

Where magnetic field due to the current I_2
in S_2 $B_2 = \mu_0 n_2 I_2$

Therefore

$$\begin{aligned} N_1 \Phi_1 &= (n_1 l) (\pi r_1^2) (\mu_0 n_2 I_2) \\ &= \mu_0 n_1 n_2 \pi r_1^2 l I_2 \end{aligned}$$

where $n_1 l$ is the total number of turns in solenoid S_1 .

Where

$$M_{12} = \mu_0 n_1 n_2 \pi r_1^2 l$$

We can easily see that

$$M_{12} = M_{21} = M \text{ (Say)}$$

if a medium of relative permeability μ_r had been present, the mutual inductance would be

$$M = \mu_r \mu_0 n_1 n_2 \pi r_1^2 l$$

It is also important to know that the mutual inductance of a pair of coils, solenoids, etc., depends on their separation as well as their relative orientation.

We have

$$N_1 \Phi_1 = MI_2$$

For currents varying

with time,

$$\frac{d(N_1 \Phi_1)}{dt} = \frac{d(MI_2)}{dt}$$

Induced emf in coil C_1 is given by

$$\varepsilon_1 = -\frac{d(N_1\Phi_1)}{dt} = -M\frac{dI_2}{dt}$$

It shows that varying current in a coil can induce emf in a neighbouring coil. The magnitude of the induced emf depends upon the rate of change of current and mutual inductance of the two coils.

Self-inductance

It is also possible that **emf** is induced in a **single** isolated coil due to change of flux through the coil by means of **varying** the current through the **same** coil. This phenomenon is called **self-induction**.

In this case, flux linkage through a coil of N turns is proportional to the current through the coil and is expressed as

$$N\Phi_B \propto I$$

$$\text{and } N\Phi_B = L I$$

where constant of proportionality L is called **self-inductance** of the coil. It is also called the **coefficient of self-induction** of the coil.

When the current is varied, the flux linked with the coil also changes and an emf is induced in the coil.

$$\varepsilon = -\frac{d(N\Phi_B)}{dt}$$

$$\varepsilon = -L\frac{dI}{dt}$$

Let us calculate the self-inductance of a long solenoid of cross sectional area A and length l , having n turns per unit length. The magnetic field due to a current I flowing in the solenoid is

$$\mathbf{B} = \mu_0 n I$$

The total flux linked with the solenoid is

$$N\Phi_B = (nl)(\mu_0 n I)(A)$$

$$= \mu_0 n^2 Al$$

where nl is the total number of turns. Thus, the self-inductance is,

$$L = \frac{N\Phi_B}{I}$$

$$= \mu_0 n^2 Al$$

(6.)

If we fill the inside of the solenoid with a material of relative permeability μ_r (for example soft iron, which has a high value of relative permeability),

then,

$$L = \mu_r \mu_0 n^2 Al$$

The self-inductance of the coil depends on its geometry and on the permeability of the medium.

The self-induced emf is also called the back emf as it opposes any change in the current in a circuit.

Physically, the **self-inductance** plays the role of **inertia**. It is the electromagnetic analogue of **mass** in mechanics.

So, work needs to be done against the back emf (ε) in establishing the current. This work done is stored as magnetic potential energy.

For the current I at an instant in a circuit, the rate of work done is

$$\frac{dW}{dt} = |\mathcal{E}| I$$

If we ignore the resistive losses and consider only inductive effect,

$$\frac{dW}{dt} = L I \frac{dI}{dt}$$

Total amount of work done in establishing the current I is

$$W = \int dW = \int_0^I L I dI$$

Thus, the energy required to build up the current I is,

$$W = \frac{1}{2} L I^2$$