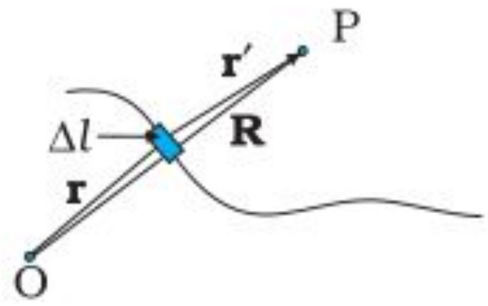
linear charge distribution

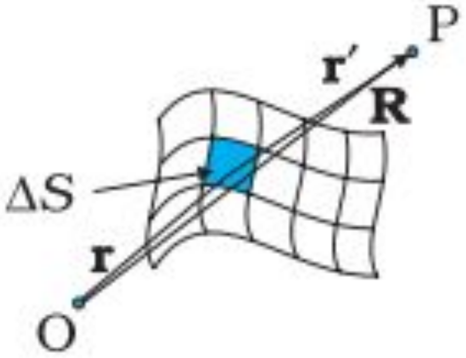


Line charge $\Delta Q = \lambda \, \Delta l$

The linear charge density λ of a wire is defined by

$$\lambda = \frac{\Delta Q}{\Delta l}$$

surface charge distribution

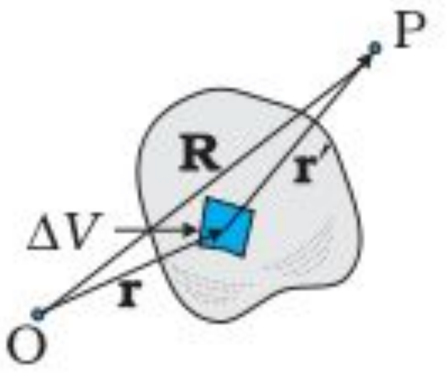


Surface charge $\Delta Q = \sigma \Delta S$

surface charge density σ at the area element by

$$\sigma = \frac{\Delta Q}{\Delta S}$$

Volume Charge distributions



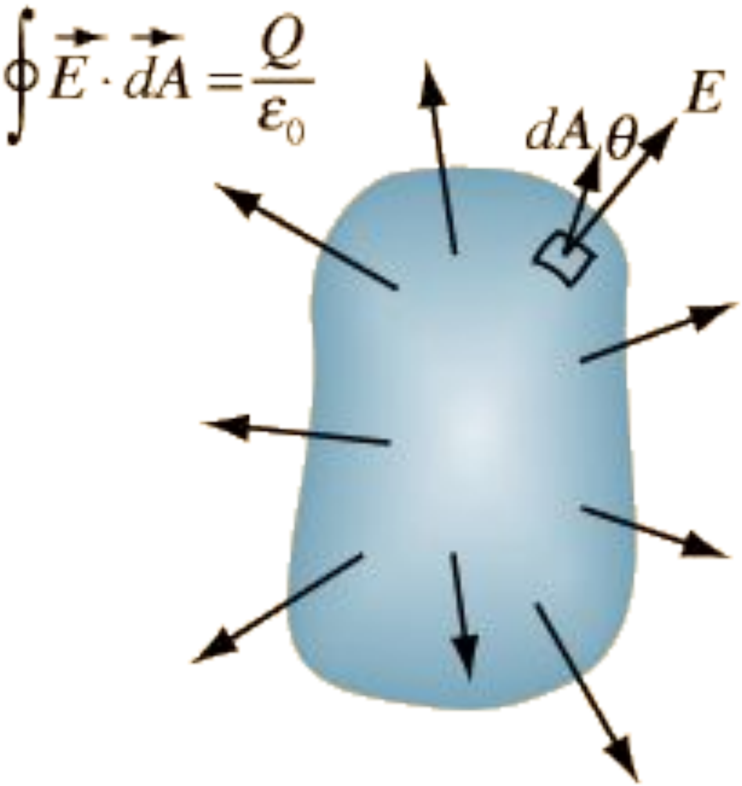
Volume charge $\Delta Q = \rho \Delta V$

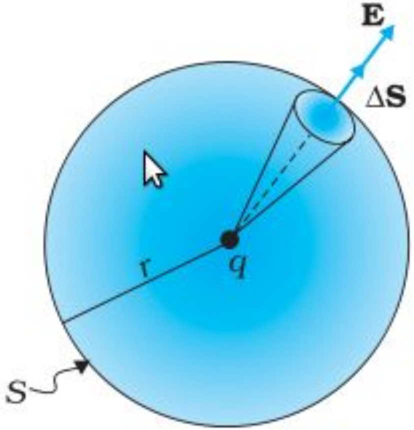
The volume charge density (sometimes simply called charge density) is defined

$$\rho = \frac{\Delta Q}{\Delta V}$$

GAUSS'S LAW

The **Surface integral** of the Electric Field of a closed surface in free space is equal to **(1/ε_0)** times the **total charge** enclosed by the surface Go to Image website





The flux through an area element ΔS is

$$\Delta\phi = \mathbf{E}\cdot\Delta\,\mathbf{S} = \frac{q}{4\pi\varepsilon_0\, r^2}\,\hat{\mathbf{r}}\cdot\Delta\mathbf{S}$$

From the figure it is clear that the area element ΔS and and the unit vector are always in the same direction. Therefore

$$\Delta\phi = \frac{q}{4\pi\varepsilon_0\, r^2}\Delta S$$

since the magnitude of a unit vector is 1.

The total flux through the sphere is obtained by adding up flux through all the different area elements:

$$\phi = \sum_{all\ \Delta S} \frac{q}{4\pi\varepsilon_0 r^2} \Delta S$$

Since each area element of the sphere is at the same distance r from the charge,

$$\phi = \frac{q}{4\pi\varepsilon_o r^2} \sum_{all\ \Delta S} \Delta S = \frac{q}{4\pi\varepsilon_0 r^2} S$$

Now S, the total area of the sphere, equals $4\pi r^2$. Thus,

$$\phi = \frac{q}{4\pi\varepsilon_0 r^2} \times 4\pi r^2 = \frac{q}{\varepsilon_0}$$

We state Gauss's law

Electric flux through a closed surface S

$$\varphi = q/\varepsilon_0$$

q = total charge enclosed by S.

Gauss's law is true for **any closed** surface, no matter what its **shape** or **size**.

The term q on the right side of Gauss's law, includes the **sum** of all charges **enclosed** by the surface. The charges may be located **anywhere** inside the surface.

The term q on the right side of Gauss's law, represents only the total charge inside S.

The surface that we **choose** for the application of Gauss's law is called the **Gaussian** surface.

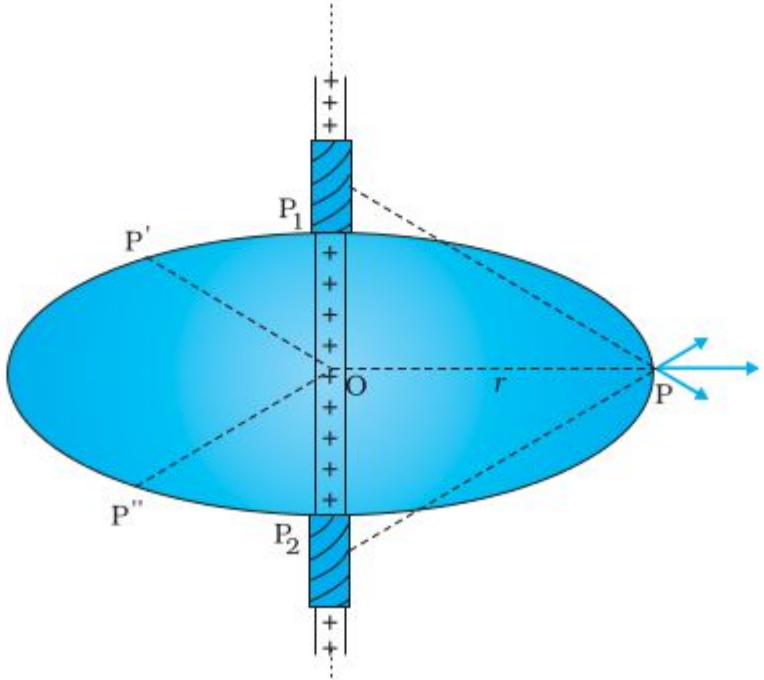
Gaussian surface can pass through a **continuous** charge distribution not pass through any **discrete** charge.

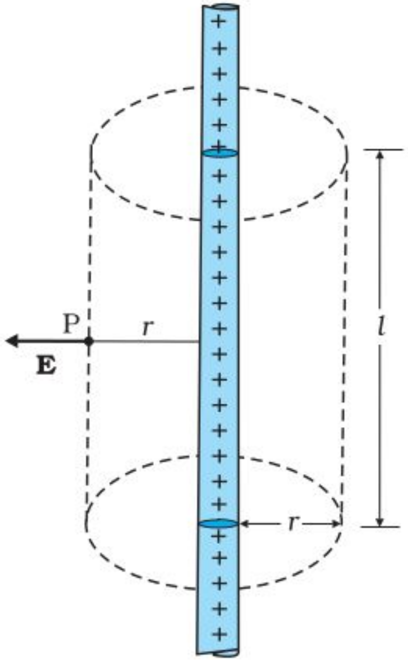
Gauss's law is often useful towards a much easier calculation of the electrostatic field when the system has some **symmetry**.

Gauss's law is based on the **inverse square** dependence on distance contained in the Coulomb's law. Any violation of Gauss's law will indicate departure from the inverse square law

APPLICATIONS OF GAUSS'S LAW

Field due to an infinitely long straight uniformly charged wire





Consider an infinitely long thin straight wire with uniform linear charge density λ. The wire is obviously an axis of symmetry

To calculate the field, imagine a cylindrical Gaussian surface.Since the field is everywhere radial, flux through the two **ends** of the cylindrical Gaussian surface is **zero**

At the **cylindrical** part of the surface, E is **normal** to the surface at every point, and its magnitude is constant, since it depends only on **r**. The surface area of the curved part is **2πrl**, where l is the length of the cylinder.

Flux through the Gaussian surface = flux through the curved cylindrical part of the surface = $E \times 2\pi rl$

The surface includes charge equal to λ l. Gauss's law then gives

$E \times 2\pi rl = \lambda l/\varepsilon_0$

i.e., $$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

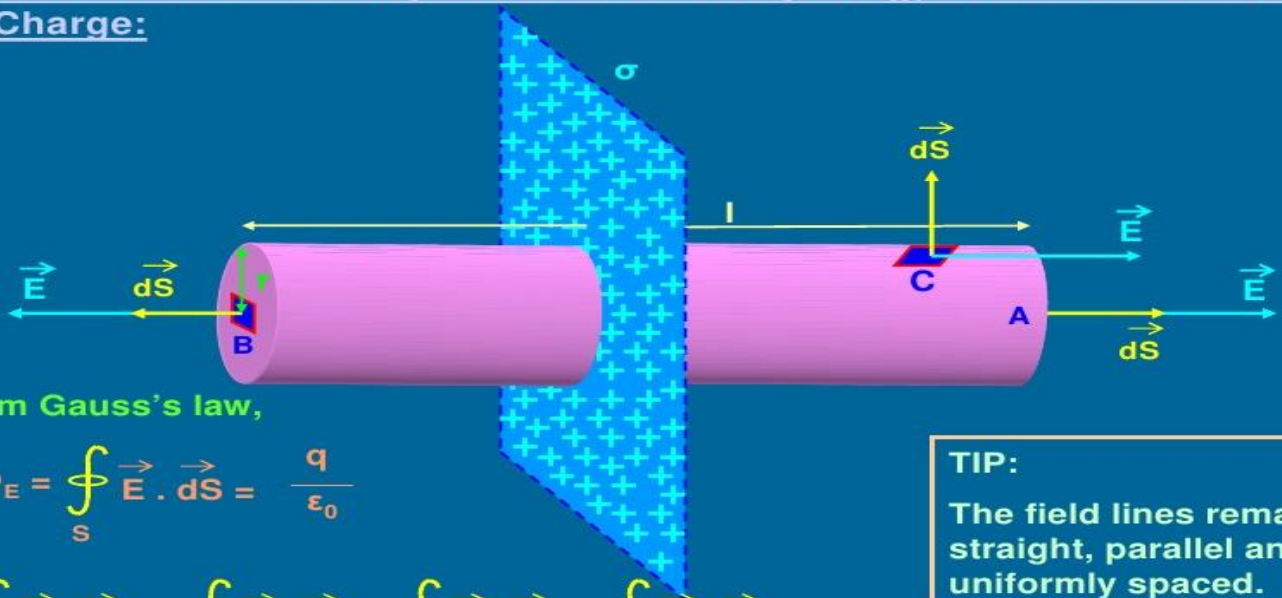
Vectorially, **E** at any point is given by

$$\mathbf{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\,\hat{\mathbf{n}}$$

where n is the radial unit vector in the plane normal to the wire passing through the point.

E is directed outward if λ is positive and inward if λ is negative

2. Electric Field Intensity due to an Infinitely Long, Thin Plane Sheet of Charge:



From Gauss's law,

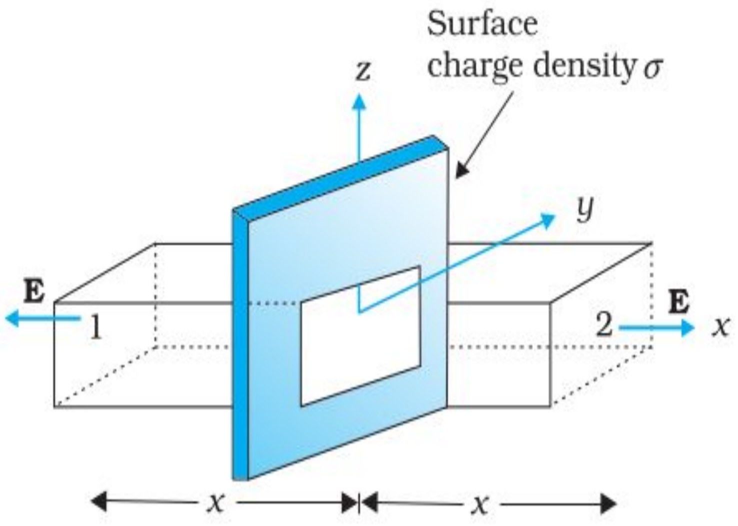
$$\Phi_E = \oint_S \vec{E} \cdot \vec{dS} = \frac{q}{\varepsilon_0}$$

TIP:

The field lines remain straight, parallel and uniformly spaced.

$$\oint_S \vec{E} \cdot \vec{dS} = \int_A \vec{E} \cdot \vec{dS} + \int_B \vec{E} \cdot \vec{dS} + \int_C \vec{E} \cdot \vec{dS}$$

$$\oint_S \vec{E} \cdot \vec{dS} = \int_A E\, dS \cos 0^\circ + \int_B E\, dS \cos 0^\circ + \int_C E\, dS \cos 90^\circ = 2E \int dS = 2E \times \pi r^2$$



Let σ be the uniform surface charge density of an infinite plane sheet .We take the x-axis normal to the given plane.

By symmetry, the electric field will not depend on y and z coordinates and its direction at every point must be parallel to the x-direction.

The unit vector **normal** to surface **1** is in **–x** direction

while the unit vector **normal** to surface **2** is in the **+x** direction

Therefore, flux **E.ΔS** through both the surfaces are equal and **add up**.

Therefore Gaussian surface for a the net flux through the Gaussian surface is **2 EA**

$2\,EA = q/\varepsilon_0$

But $q = \sigma A$

$2\,EA = \sigma A/\varepsilon_0$

or, $E = \sigma/2\varepsilon_0$

Vectorically,

$$\mathbf{E} = \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{n}}$$

where n is a unit vector normal to the plane and going away from it.

Field due to a uniformly charged thin spherical shell