Circular current loop as a magnetic dipole

In this section, we shall consider the elementary magnetic element:

the current loop.

We shall show that the magnetic field (at large distances) due to current in a circular current loop is very similar in behavior to the electric field of an electric dipole.

The Magnetic field on the axis of a circular loop, of a radius R, carrying a steady current **I** is given by

$$B = \frac{\mu_0 I R^2}{2\left(x^2 + R^2\right)^{3/2}}$$

Its direction is along the axis and x is the distance along the axis from the centre of the loop.

For $x >> R$, we may drop the R^2 term in the denominator, then

$$B = \frac{\mu_0 R^2}{2x^3}$$

If the area of the loop $A = \pi R^2$.

Thus,

$$B = \frac{\mu_0 I A}{2\pi x^3}$$

But the magnetic moment m = IA

$$\mathbf{B} \simeq \frac{\mu_0 \mathbf{m}}{2\pi x^3}$$

$$= \frac{\mu_0}{4\pi} \frac{2\mathbf{m}}{x^3}$$

This is similar to

$$\mathbf{E} = \frac{2\mathbf{p}_e}{4\pi \varepsilon_0 x^3}$$

in electrstatics

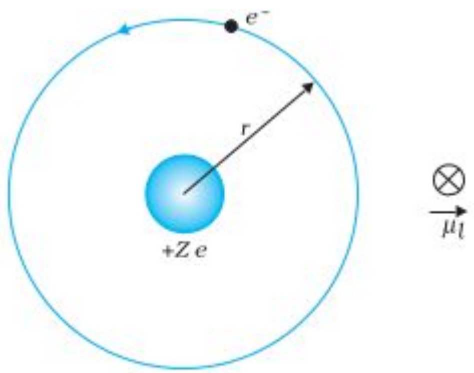
We have shown that a current loop

(i) produces a magnetic field and behaves like a magnetic dipole at large distances, and

(ii) is subject to torque like a magnetic needle. This led Ampere to suggest that all magnetism is due to circulating currents. This seems to be partly true and no magnetic monopoles have been seen so far.

However, elementary particles such as an electron or a proton also carry an intrinsic magnetic moment, not accounted by circulating currents

The magnetic dipole moment of a revolving electron



The electron of charge (–e) performs uniform circular motion around a stationary heavy nucleus of charge +Ze. This constitutes a current I

$$I = \frac{e}{T}$$

T is the time period

of revolution

Let r be the orbital radius of the electron, and v the orbital speed. Then, $T = \frac{2\pi r}{v}$

Substituting , we have I = **ev/2πr.**

There will be a magnetic moment, usually denoted by μ_l, associated with this circulating current. $\mu_l = iA = I\pi r^2 = evr/2.$

Multiplying and dividing the right-hand side of the above expression by the electron mass $\mathbf{m_e}$, we have,

$$\mu_l = \frac{e}{2m_e}(m_e v r)$$

$$= \frac{e}{2m_e} l$$

Here, l is the magnitude of the angular momentum of the electron about the central nucleus (“orbital” angular momentum).

Vectorially, $\boldsymbol{\mu}_l = -\frac{e}{2m_e}\mathbf{l}$

The **negative** sign indicates that the angular momentum of the electron is **opposite** in direction to the magnetic moment.

$\frac{\mu_l}{l} = \frac{e}{2m_e}$ is called the gyromagnetic ratio and is a constant.

Its value is 8.8×10^{10} C /kg for an electron, Bohr hypothesised that the angular momentum assumes a discrete set

of values, namely,

$$l = \frac{nh}{2\pi}$$

where n is a natural number, n = 1, 2, 3, ... and h is a constant named after Max Planck (Planck's constant) with a value

$$h = 6.626 \times 10^{-34} \text{ J s.}$$

Take the value n = 1, we have

$$(\mu_l)_{min} = \frac{e}{4\pi m_e} h$$

$$= \frac{1.60 \times 10^{-19} \times 6.63 \times 10^{-34}}{4 \times 3.14 \times 9.11 \times 10^{-31}}$$

$$= 9.27 \times 10^{-24} \text{ Am}^2$$

where the subscript ‘min’ stands for minimum. This value is called the **Bohr magneton**

Besides the orbital moment, the electron has an intrinsic magnetic moment, . It is called the **spin magnetic** moment