CAPACITORS AND CAPACITANCE

The electric field in the region between the conductors is proportional to the charge Q.

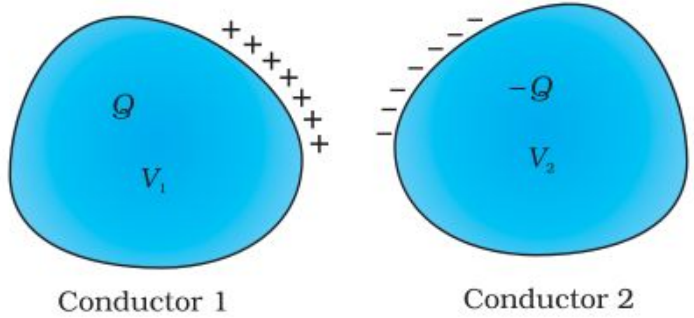


FIGURE 2.24 A system of two conductors separated by an insulator forms a capacitor.

potential difference V is the work done per unit positive charge in taking a small test charge from the conductor 2 to 1 against the field.

A system of two conductors Consequently, V is also proportional to Q, and separated by an insulator forms a capacitor.

the ratio **Q/V** is a constant:

$$C = \frac{Q}{V}$$

The constant C is called the capacitance of the capacitor. C is independent of Q or V,

The capacitance C depends only on the geometrical configuration (shape, size, separation) of the system of two conductors.

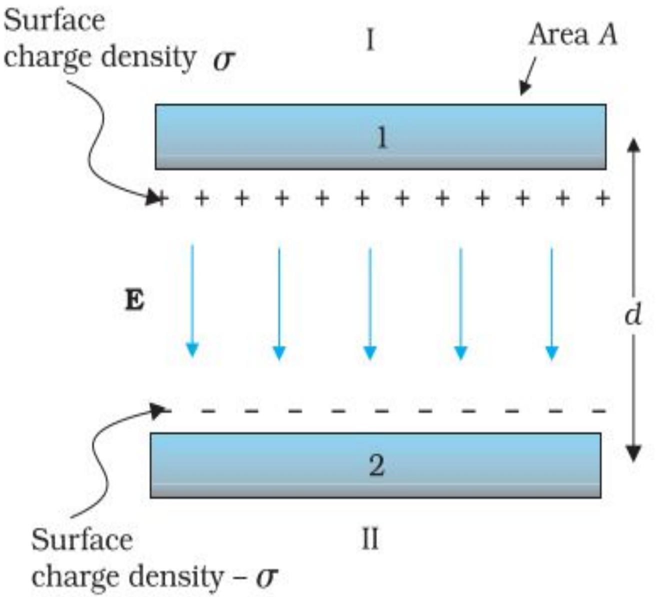
The SI unit of capacitance is 1 farad

The maximum electric field that a dielectric medium can withstand without break-down (of its insulating property) is called its **dielectric strength**;

for air it is about 3×10^6 Vm–1.

For a separation between conductors of the order of 1 cm or so, this field corresponds to a potential difference of 3×10^4 V between the conductors

THE PARALLEL PLATE CAPACITOR



Let A be the area of each plate and d the separation between them.

Plate 1 has surface charge density σ = Q/A and plate 2 has a surface charge density –σ.

Outer region I

(region above the plate 1),

$$E = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$$

Outer region II

(region below the plate 2),

$$E = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$$

In the inner region between the plates 1 and 2, the electric fields due to the two charged plates add up, giving

$$E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$$

EFFECT OF DIELECTRIC ON CAPACITANCE

we have two large plates, each of area A, separated by a distance d. The charge on the plates is ±Q, corresponding to the charge density ±σ (with σ = Q/A).

When there is vacuum between the plates,

$$E_0 = \frac{\sigma}{\varepsilon_0}$$

and the potential difference V0 is

$$\mathbf{V_0 = E_0 d}$$

The capacitance C_0 in this case is

$$C_0 = \frac{Q}{V_0} = \varepsilon_0 \frac{A}{d}$$

Consider next a dielectric inserted between the plates fully occupying the intervening region.

The electric field in the dielectric then corresponds to the case when the net surface charge density on the plates is ±(σ – σp).

That is,

$$E = \frac{\sigma - \sigma_P}{\varepsilon_0}$$

so that the potential difference across the plates is

$$V = Ed = \frac{\sigma - \sigma_P}{\varepsilon_0} d$$

For linear dielectrics, we expect σ_p to be proportional to E_0, i.e., to σ.

$$V = Ed = \frac{\sigma - \sigma_P}{\varepsilon_0} d$$

where K is a constant characteristic of the dielectric.

$$\sigma - \sigma_P = \frac{\sigma}{K}$$

Clearly, K > 1. We then have

$$V = \frac{\sigma d}{\varepsilon_0 K} = \frac{Qd}{A\varepsilon_0 K}$$

The capacitance C, with dielectric between the plates, is then

$$C = \frac{Q}{V} = \frac{\varepsilon_0 KA}{d}$$

The product $\varepsilon_0 K$ is called the permittivity of the medium and is denoted by ε

ie $\varepsilon = \varepsilon_0 K$

For vacuum $K = 1$ and $\varepsilon = \varepsilon_0$; ε_0 is called the permittivity of the vacuum.

The dimensionless ratio is called the dielectric constant of the substance

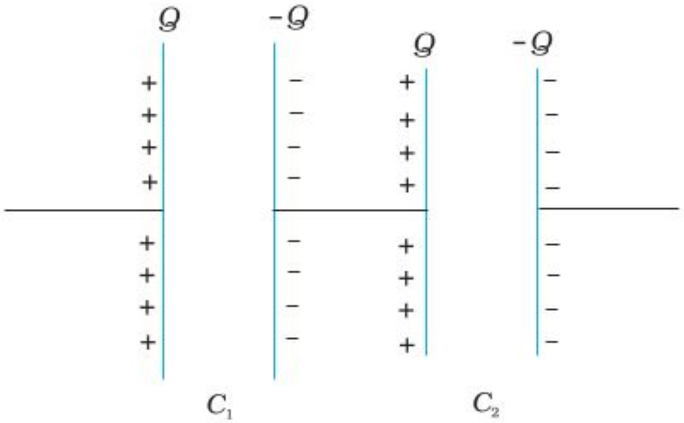
$$K = \frac{\varepsilon}{\varepsilon_0}$$

Thus, the dielectric constant of a substance is the factor (>1) by which the capacitance increases from its vacuum value, when the dielectric is inserted fully between the plates of a capacitor.

$$K = \frac{C}{C_0}$$

COMBINATION OF CAPACITORS

Capacitors in series



The total potential drop **V** across the combination is t

potential drops V_1 and V_2 across C_1 and C_2, respectively.

$$V = V_1 + V_2 = \frac{Q}{C_1} + \frac{Q}{C_2}$$

i.e., $$\frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2},$$

Now we can regard the combination as an effective capacitor with charge Q and potential difference V. The effective capacitance of the combination is

$$C = \frac{Q}{V}$$

Comparing the above equations we get

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

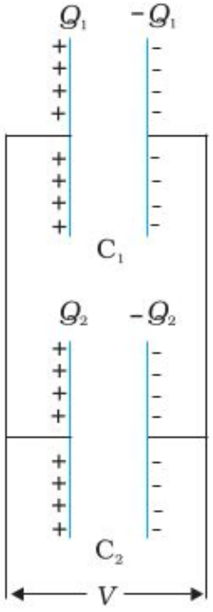
for n capacitors arranged in series

$$V = V_1 + V_2 + \ldots + V_n = \frac{Q}{C_1} + \frac{Q}{C_2} + \ldots + \frac{Q}{C_n}$$

and

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots + \frac{1}{C_n}$$

Capacitors in parallel



In this case, the same potential difference is applied across both the capacitors. But the plate charges ($\pm Q_1$) on capacitor 1 and the plate charges ($\pm Q_2$) on the capacitor 2 are not necessarily the same:

$$Q_1 = C_1 V,\ Q_2 = C_2 V$$

and

$$Q = Q_1 + Q_2$$

$$Q = CV = C_1 V + C_2 V$$

The effective capacitance C is $C = C_1 + C_2$

The general formula for effective capacitance C for parallel combination of n capacitors

$$Q = Q_1 + Q_2 + \ldots + Q_n$$

i.e., $CV = C_1V + C_2V + \ldots C_nV$

which gives

$$C = C_1 + C_2 + \ldots C_n$$

ENERGY STORED IN A CAPACITOR

Work has to be done to store charges in a capacitor. This work done will be stored in the capacitor as electrostatic Potential energy

This energy can be recovered when the capacitor is allowed to discharge let V be the PD and Q be the charge on a capacitor. when an additional charge dQ is given to the plate,

work done dW=VdQ

But $V = \frac{Q}{C}$ Therefore

$$dW = \frac{Q}{C} dQ$$

The total workdone to charge the capacitor from zero to Q is given by

$$w = \int dW = \int_0^Q \frac{Q}{C} \, dQ$$

ie

$$W = \frac{Q^2}{2C}$$

This is the electrostatic PE stored in the capacitor

But Q = CV then $U = W = \frac{1}{2}CV^2$

OR $U = \frac{1}{2}QV$

Energy Density

Energy density is the energy stored per unit volume of the capacitor

$$u = U/\text{Volume} = \frac{\frac{1}{2} C V^2}{A d}$$

$$= \frac{\frac{1}{2} \frac{\varepsilon_0 A}{d} V^2}{A d}$$

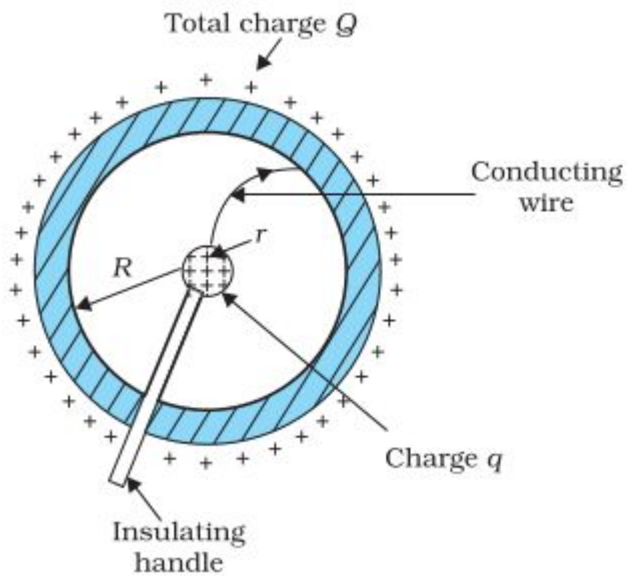
$$u = \frac{1}{2}\frac{\varepsilon_0 V^2}{d^2} = \frac{1}{2}\varepsilon_0 E^2$$

$$u = (1/2)\varepsilon_0 E^2$$

VAN DE GRAAFF GENERATOR

This is a machine that can build up high voltages of the order of a few million volts. The resulting large electric fields are used to accelerate charged particles (electrons, protons, ions) to high energies needed for experiments to probe the small scale structure of matter

Taking both charges q and Q into account we have for the total potential V and the potential difference the values



$$V(R)=\frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R}+\frac{q}{R}\right)$$ and $$V(r)=\frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R}+\frac{q}{r}\right)$$

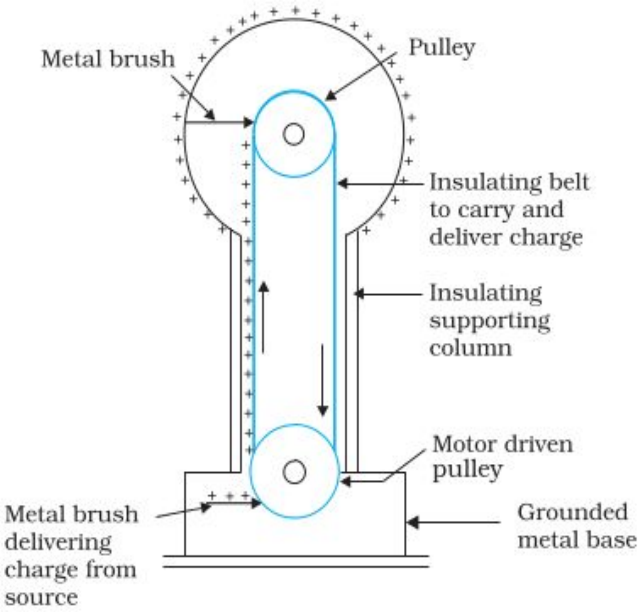
Clearly V(r) > V(R) and the charges moves from higher potential to lower potential

The PD $V(r) - V(R) = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{R}\right)$ is positive and

is independent of Q

This means that if we now connect the smaller and larger sphere by a wire, the charge q moves from smaller sphere to larger sphere

A large spherical conducting shell (of few metres radius) is supported at a of Van de Graaff generator.



A long narrow endless belt insulating material, like rubber or silk, is wound around two pulleys

–one at ground level, one at the centre of the shell. This belt is kept continuously moving by a motor driving the lower pulley.

It continuously carries positive charge, sprayed on to it by a brush at ground level, to the top. There it transfers its positive charge to another conducting brush connected to the large shell.

Thus positive charge is transferred to the shell, where it spreads out uniformly on the outer surface.