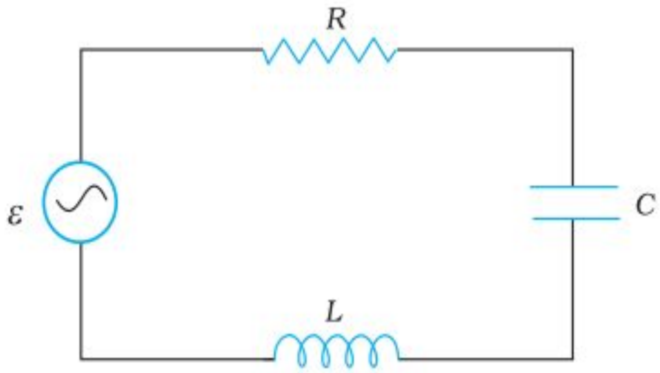
AC VOLTAGE APPLIED TO A SERIES LCR CIRCUIT

we take the voltage of the source to b

$$v = v_m \sin \omega t.$$

If q is the charge on the capacitor and



current, at time t, we have, from Kirchhoff's loop

rule:

$$L\frac{di}{dt} + iR + \frac{q}{C} = v$$

Phasor-diagram solution

we see that the resistor, inductor and capacitor are in **series**. Therefore, the ac current in each element is the same at any time, having the same amplitude and phase. Let it be **$i = i_m \sin(\omega t+\varphi)$**

where φ is the phase difference between the voltage across the source and the current in the circuit

Let I be the phasor representing the current in the circuit. Further, let V_L, V_R, V_C, and V represent the voltage across the inductor, resistor, capacitor and the source, respectively.

we know that V_R is parallel to I, V_C is π/2 behind I

and V_L is π/2 ahead of I.

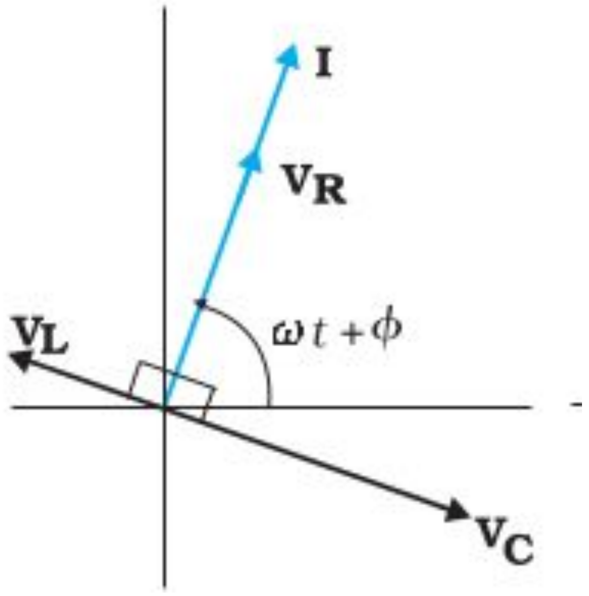
VL, VR, VC and I are shown with

apppropriate phase- relations.

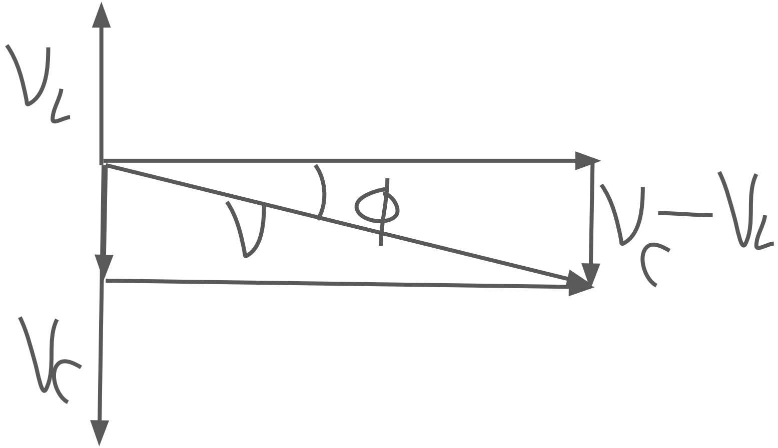
The length of these phasors or the

amplitude of V_R, V_C and V_L are:

$v_{Rm} = i_m R$, $v_{Cm} = i_m X_C$, $v_{Lm} = i_m X_L$



In the vector diagram the resultant voltage is given by



$$V^2 = V_R^2 + (V_c - V_L)^2$$

$$I^2Z^2 = I^2R^2 + I^2(X_c - X_L)^2$$

$$\text{Or } Z^2 = R^2 + (X_c - X_L)^2$$

where $Z = \sqrt{R^2 + (X_C - X_L)^2}$ is then Impedance of the LCR circuit

Since phasor I is always parallel to phasor V_R,

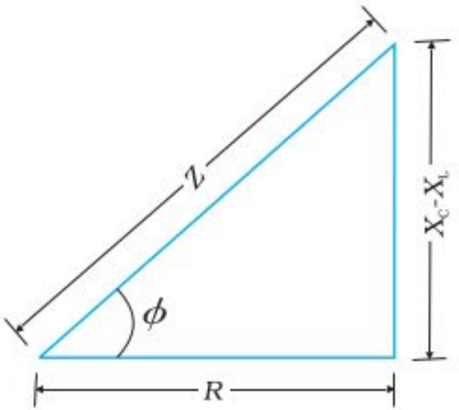
the phase angle φ is the angle between V_R and V

and can be determined from the figure

as

$$\tan\phi = \frac{v_C - v_L}{v_R}$$

$$\tan\phi = \frac{X_C - X_L}{R}$$

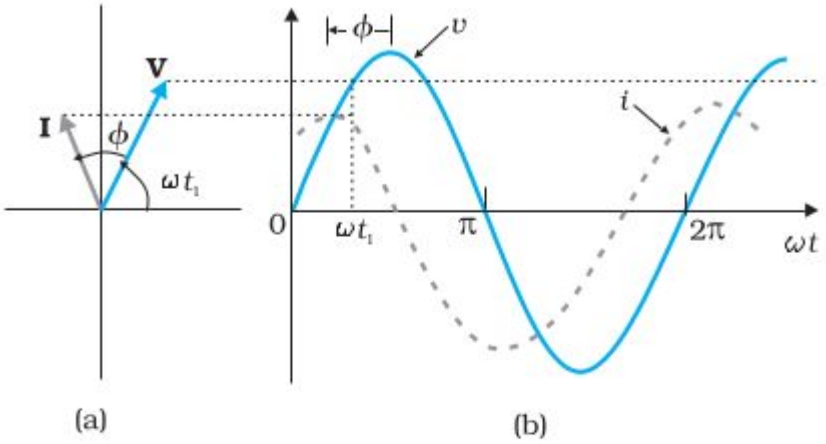


If $X_C > X_L$, φ is positive and the circuit is predominantly capacitive.

Consequently, the current in the circuit leads the source voltage.

If $X_C < X_L$, φ is negative and the circuit is predominantly inductive. Consequently, the current in the circuit lags the source voltage.

The phasor diagram and variation of v and i with ω t for the case **$X_C > X_L$.**



Resonance

The phenomenon of resonance is common among systems that have a tendency to **oscillate** at a **particular frequency**. This frequency is called the system's **natural frequency**. If such a system is **driven** by an energy source at a frequency that is **near the natural frequency**, the **amplitude** of oscillation is found to be **large**.

For an RLC circuit driven with voltage of amplitude v_m and frequency ω, we found that the current amplitude

$$i_m = \frac{v_m}{Z} = \frac{v_m}{\sqrt{R^2 + (X_C - X_L)^2}}$$

is given by

with Xc = 1/ωC and XL = ω L . So if ω is varied, then at a particular frequency

ω_0, $X_c = X_L$, and the impedance is minimum Z = R + 0 = R . This

frequency is called the resonant frequency:

Resonant circuits have a variety

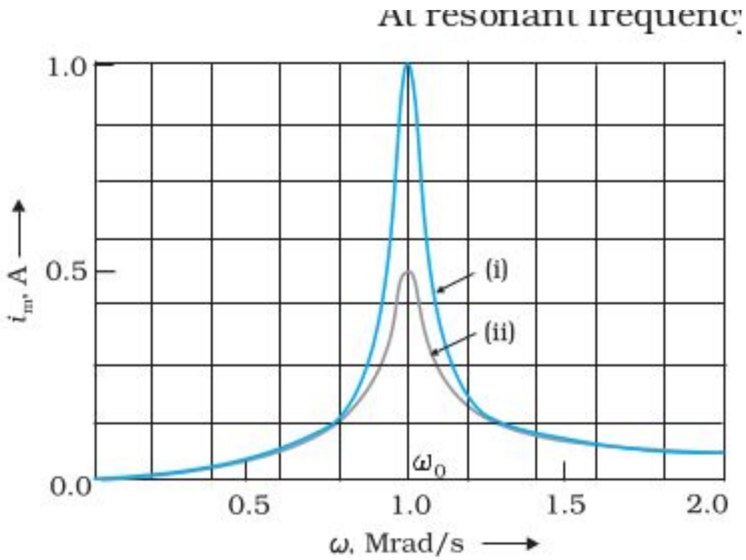
of applications, for example,

in the tuning mechanism of a radio

or a TV set. The antenna of a radio

accepts signals from many broadcasting

stations



Sharpness of resonance

The amplitude of the current in the series LCR circuit is given by

$$i_m = \frac{v_m}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

It is maximum

at $\omega = \omega_0 = 1/\sqrt{LC}.$

maximum value of current is $i_m^{\max} = v_m / R.$

For values of ω other than ω0, the amplitude of the current is less than the maximum value. Suppose we choose a value of ω for which the current amplitude is **1/ 2** times its maximum value.

At this value, the power dissipated by the circuit becomes half. The frequencies of half power points are

$$\omega_1 = \omega_0 + \Delta\omega$$

$$\omega_2 = \omega_0 - \Delta\omega$$

The difference $\boldsymbol{\omega_1 - \omega_2 = 2\Delta\omega}$ is often called the **bandwidth** of the circuit.

Sharpness at resonance (Q factor) can be defined as

$Q = X_L/R = X_C / R$

or $Q = f_o / f_2 - f_1$

POWER IN LCR AC CIRCUIT: THE POWER FACTOR

We have seen that a voltage $\mathbf{v = v_m \sin\omega t}$ applied to a series RLC circuit drives a current in the circuit given by $\mathbf{i = i_m \sin(\omega t + \varphi)}$ where

$$i_m = \frac{v_m}{Z} \quad \text{and} \quad \phi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$$

Therefore, the instantaneous power p supplied by the source is

$$p = v\,i = (v_m \sin\omega t) \times [i_m \sin(\omega t + \phi)]$$

$$= \frac{v_m i_m}{2}[\cos\phi - \cos(2\omega t + \phi)]$$

$T\cos(2\omega t + \varphi)$