

AC VOLTAGE APPLIED TO AN INDUCTOR

Thus, the circuit is a purely inductive ac circuit.

Let the voltage across the source be

$$v = v_m \sin \omega t.$$

Using the Kirchhoff's loop rule,

$$v - L \frac{di}{dt} = 0$$

where the second term is the self-induced Faraday emf in the inductor; and L is the self-inductance of the inductor



Combining above two equations

$$\frac{di}{dt} = \frac{v}{L} = \frac{v_m}{L} \sin \omega t$$

Integrating on both sides

$$\int \frac{di}{dt} dt = \frac{v_m}{L} \int \sin(\omega t) dt$$

and get,

$$i = -\frac{v_m}{\omega L} \cos(\omega t) + \text{constant}$$

the integration constant is zero

But

$$-\cos(\omega t) = \sin\left(\omega t - \frac{\pi}{2}\right), \text{ we have}$$

$$i = i_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

Where $i_m = \frac{v_m}{\omega L}$ is the Amplitude of the Current

The quantity ωL is analogous to the resistance and is called inductive reactance, denoted by X_L :

$$X_L = \omega L$$

The amplitude of the current is, then $i_m = \frac{v_m}{X_L}$

A comparison of equations of current and the voltage shows that the current lags the voltage by $\pi/2$ or one-quarter (1/4) cycle

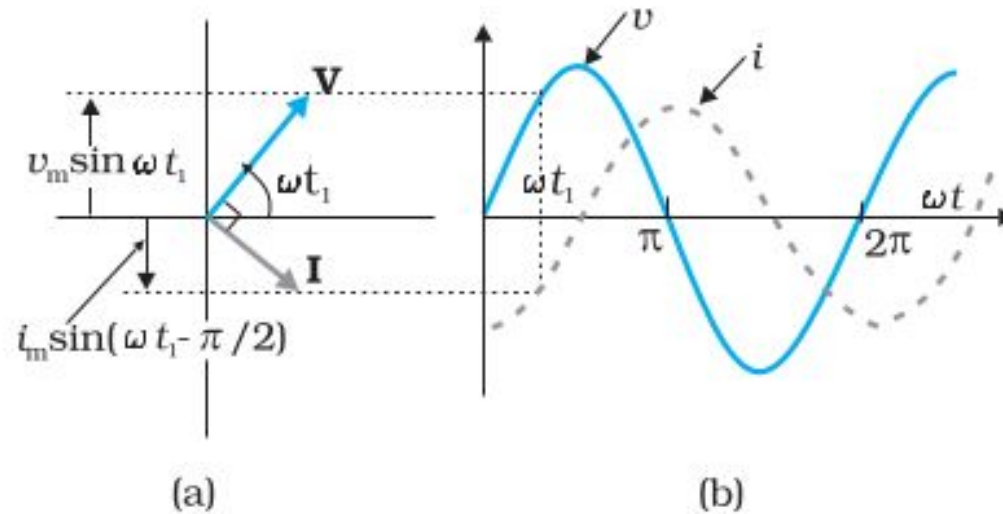
The dimension of inductive reactance is the same as that of resistance and its SI unit is ohm (Ω). The inductive reactance limits the current in a purely inductive circuit in the same way as the resistance limits the current in a purely resistive circuit.

The inductive reactance is directly proportional to the inductance and to the frequency of the current.

Inductance allows a high reactance to AC and allows DC to pass.

The current phasor I is $\pi/2$ behind the voltage phasor V .

When rotated with frequency ω counter-clockwise, they generate the voltage and current



The instantaneous power supplied to the inductor is

$$\begin{aligned} p_L &= i v = i_m \sin\left(\omega t - \frac{\pi}{2}\right) \times v_m \sin(\omega t) \\ &= -i_m v_m \cos(\omega t) \sin(\omega t) \\ &= -\frac{i_m v_m}{2} \sin(2\omega t) \end{aligned}$$

So, the average power over a complete cycle is

$$\begin{aligned} P_L &= \left\langle -\frac{i_m v_m}{2} \sin(2\omega t) \right\rangle \\ &= -\frac{i_m v_m}{2} \langle \sin(2\omega t) \rangle = 0, \end{aligned}$$

since the average of $\sin(2\omega t)$

over a complete cycle is zero. Thus, the average power supplied to an inductor over one complete cycle is zero (wattless current)