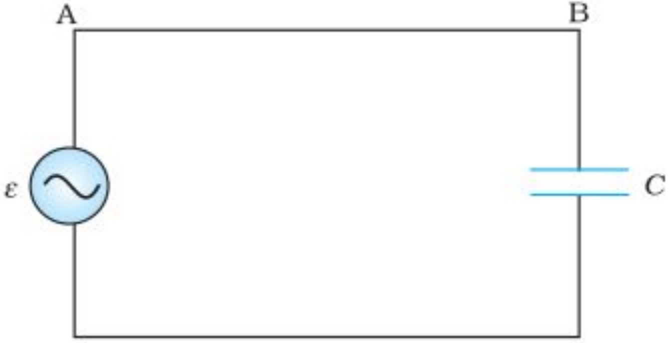
AC VOLTAGE APPLIED TO A CAPACITOR

When a capacitor is connected to a voltage source in a dc circuit, current will flow for the short time required to charge the capacitor.



As charge accumulates on the capacitor plates, the voltage across them increases, opposing the current.

That is, a capacitor in a dc circuit will limit or oppose the current as it charges. When the capacitor is fully charged, the current in the circuit falls to zero.

Let q be the charge on the capacitor at any time t. The instantaneous voltage v across the capacitor is

$$v = \frac{q}{C}$$

From the Kirchhoff's loop rule, the voltage across the source and the capacitor are equal,

$$v_m \sin \omega t = \frac{q}{C}$$

To find the current, we use the relation

$$i = \frac{dq}{dt}$$

$$i = \frac{d}{dt}\left(v_m C \sin \omega t\right) = \omega C v_m \cos(\omega t)$$

Using the relation, $\cos(\omega t) = \sin\left(\omega t + \frac{\pi}{2}\right)$, we have

$$i = i_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

where the amplitude of the oscillating current is $\mathbf{i_m = \omega C v_m}$. We can rewrite it as

$$i_m = \frac{v_m}{(1/\omega C)}$$

we find that (**$1/\omega C$**) plays the role of resistance. It is called capacitive reactance and is denoted by $\mathbf{X_c}$,

$$X_c = 1/\omega C$$

so that the amplitude of the cu $i_m = \frac{v_m}{X_C}$

The dimension of capacitive reactance is ohm (Ω). The capacitive reactance limits the amplitude of the current in a purely capacitive circuit in the same way as the resistance limits the current in a purely resistive circuit. But **capacitive reactance** is **inversely** proportional to the frequency and the capacitance.

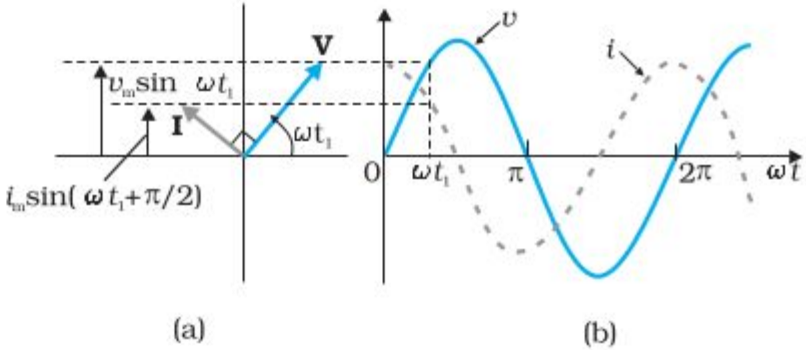
In a purely capacitative circuit the current is π/2 ahead of voltage. Here the current phasor I is **π/2 ahead** of the voltage phasor V as they rotate

counterclockwise.

We see that the

current reaches its maximum

value earlier than the voltage by one-fourth of a period



The instantaneous power supplied to the capacitor is

$$p_c = i\,v = i_m \cos(\omega t) v_m \sin(\omega t)$$

$$= i_m v_m \cos(\omega t) \sin(\omega t)$$

$$= \frac{i_m v_m}{2} \sin(2\omega t)$$

So, as in the case of an inductor, the average power

$$P_C = \left\langle \frac{i_m v_m}{2} \sin(2\omega t) \right\rangle = \frac{i_m v_m}{2} \langle \sin(2\omega t) \rangle = 0$$

Since $\langle \sin(2\omega t) \rangle = 0$ over a complete cycle

Thus, we see that in the case of an **inductor**, the current **lags** the voltage by π/2 and in the case of a **capacitor**, the current **leads** the voltage by π/2. The current and the voltage are **same** phase in the **resistor**